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Thermal oscillation and resonance in dual-phase-lagging heat conduction

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Abstract

We examine thermal oscillation and resonance (with respect to time) described by the dual-phase-lagging heat-conduction equations analytically. Conditions and features of underdamped, critically damped and overdamped oscillations are obtained and compared with those described by the classical parabolic heat-conduction equation and the hyperbolic heat-conduction equation. Also derived is the condition for the thermal resonance. Both the underdamped oscillation and the critically damped oscillation cannot appear if the phase lag of the temperature gradient τ_T is larger than that of the heat flux τ_q. The modes of underdamped thermal oscillation are limited to a region fixed by two relaxation distances defined by $\sqrt{\alpha\tau_T}(\sqrt{(\tau_q/\tau_T)} + \sqrt{(\tau_q/\tau_T) - 1})$ and $\sqrt{\alpha\tau_T}(\sqrt{(\tau_q/\tau_T)} - \sqrt{(\tau_q/\tau_T) - 1})$ for the case of $\tau_T > 0$, and by one relaxation distance $2\sqrt{\alpha\tau_q}$ for the case of $\tau_T = 0$. Here α is the thermal diffusivity of the medium. © 2002 Elsevier Science Ltd. All rights reserved.

1. Introduction

The high-rate heating is developing rapidly due to the advancement of high-power short-pulse laser technologies [1–5]. The phonon–electron interaction model [6], the phonon scattering model [7,8], and the dual-phase-lagging model [9,10] are developed in examining energy transport involving the high-rate heating in which the non-equilibrium thermodynamic transition and the microstructural effect become important associated with shortening of the response time. These models lead to the dual-phase-lagging heat-conduction equation [10]. In addition to its application in the ultrafast pulse-laser heating, the dual-phase-lagging heat-conduction equation also arises in describing and predicting phenomena such as temperature pulses propagating in superfluid liquid helium, non-homogeneous lagging response in porous media, thermal lagging in amorphous materials, and effects of material defects and thermomechanical coupling [10]. A study of the behavior of temperature field based on the dual-phase-lagging heat-conduction equation is thus of considerable importance in understanding and applying these rapidly emerging technologies.

The dual-phase-lagging heat-conduction equation was shown to be both admissible within the framework of the second law of the extended irreversible thermodynamics [10] and well posed in a finite region of n-dimension ($n \geqslant 1$) under Dirichlet, Neumann or Robin boundary conditions [11,12]. Solutions of one-dimensional (1D) heat conduction were obtained in [9,13–18] for some specific initial and boundary conditions. Wang and Zhou [19] developed methods of measuring the phase lags of the heat flux and the temperature gradient and obtained analytical solutions for regular 1D, 2D and 3D heat-conduction domains under essentially arbitrary initial and boundary conditions. The solution structure theorems were also developed for both mixed and Cauchy problems of dual-phase-lagging heat-conduction equations [11,19] by extending those for the hyperbolic heat conduction [20]. These theorems express contributions (to the temperature field) of the initial temperature distribution and the source term by that of the initial time-rate change of the temperature, uncover the structure of temperature field and considerably simplify the development of solutions. The thermal features

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Nomenclature

C_1	relaxation distance, m
C_2	relaxation distance, m
f_c	critical damping coefficient, s^{-1}
f_m	damping coefficient, s^{-1}
k	thermal conductivity, m^{-1} K^{-1}
l	length, m
S	volumetric heat source, W m^{-3}
T	temperature, K
t	time, s
x	coordinate, m
α	thermal diffusivity, m^2 s^{-1}
ζ_m	damping ratio parameter
η_m	non-dimensional parameter
τ_q	phase lag of heat flux vector, s
τ_T	phase lag of temperature gradient, s
ϕ	initial temperature distribution, K
ψ	initial time-rate change of temperature, K s^{-1}
Ω	frequency of heat source, s^{-1}
ω_m	modal frequency, s^{-1}

described by the dual-phase-lagging heat-conduction equations have, however, not been sufficiently addressed. In particular, the conditions and features of thermal oscillation and resonance and their difference with those in the classical and hyperbolic heat conductions are not available. This stimulates the present work to examine the thermal oscillation and resonance in dual-phase-lagging heat conductions analytically.

2. Thermal oscillation

Without losing the generality, consider the 1D initial-boundary value problem of dual-phase-lagging heat conduction

$$\begin{aligned} &\frac{1}{\alpha}\left(\frac{\partial T}{\partial t}+\tau_q\frac{\partial^2 T}{\partial t^2}\right)=\frac{\partial^2 T}{\partial x^2}+\tau_T\frac{\partial^3 T}{\partial t\,\partial x^2}\\ &\quad+\frac{1}{k}\left(S+\tau_q\frac{\partial S}{\partial t}\right), \quad (0,l)\times(0,+\infty),\\ &T(0,t)=T(l,t)=0,\\ &T(x,0)=\phi(x), \quad T_t(x,0)=\psi(x), \end{aligned} \tag{1}$$

whose solution represents the temperature distribution in an infinitely wide slab of thickness l. Here t is the time, T the temperature, α the thermal diffusivity of the medium, S the volumetric heat source, ϕ and ψ are given functions, τ_T and τ_q are the phase lags of the temperature gradient and heat flux vector, respectively. [1]

For a free thermal oscillation, $S=0$. By taking the boundary conditions into account, let

$$T(x,t)=\sum_{m=1}^{\infty}\Gamma_m(t)\sin\beta_m x, \tag{2}$$

where

$$\beta_m=\frac{m\pi}{l}.$$

Using the Fourier sine series to express ϕ and ψ as

$$\phi(x)=\sum_{m=1}^{\infty}\phi_m\sin\beta_m x \tag{3}$$

and

$$\psi(x)=\sum_{m=1}^{\infty}\psi_m\sin\beta_m x, \tag{4}$$

where

$$\phi_m=\frac{2}{l}\int_0^l\phi(\xi)\sin\beta_m\xi\,d\xi,$$

and

$$\psi_m=\frac{2}{l}\int_0^l\psi(\xi)\sin\beta_m\xi\,d\xi.$$

A substitution of (2)–(4) into (1) yields, by making use of the orthogonality of $\sin\beta_m x$ $(m=1,2,\ldots)$,

$$\tau_q\ddot{\Gamma}_m+\left(1+\alpha\tau_T\beta_m^2\right)\dot{\Gamma}_m+\beta_m^2\alpha\Gamma_m=0, \tag{5}$$

$$\Gamma_m(0)=\phi_m, \quad \dot{\Gamma}_m(0)=\psi_m. \tag{6}$$

Introduce the damping coefficient f_m by

$$f_m=\frac{1}{\tau_q}+\tau_T\omega_m^2$$

and the natural frequency coefficient ω_m by

$$\omega_m^2=\frac{\alpha\beta_m^2}{\tau_q}.$$

Eq. (5) reduces to

$$\ddot{\Gamma}_m+f_m\dot{\Gamma}_m+\omega_m^2\Gamma_m=0. \tag{7}$$

The solution of Eq. (7) can be readily obtained by the method of undetermined coefficients as

$$\Gamma_m(t)=b\,e^{\lambda t} \tag{8}$$

[1] We restrict our study, in the present work, to the case of constant τ_T and τ_q. The effects of temperature-dependent properties on the system characteristics are the topic of future study.

with λ as a coefficient to be determined. Substituting Eq. (8) into Eq. (7) leads to

$$\lambda^2 + f_m\lambda + \omega_m^2 = 0, \tag{9}$$

which has solutions λ_1, λ_2:

$$\lambda_{1,2} = -\frac{f_m}{2} \pm \sqrt{\Delta}. \tag{10}$$

Here Δ is the discriminant of Eq. (9) and is defined by

$$\Delta = \left(\frac{f_m}{2}\right)^2 - \omega_m^2.$$

Therefore, a positive, negative and vanished discriminant yields two distinct real λ_1, λ_2, two complex conjugate λ_1, λ_2, and two equal real λ_1, λ_2, respectively. The critical damping coefficient f_{mc} is referred to the damping coefficient at a fixed ω_m and $\Delta = 0$. Therefore,

$$f_{mc} = 2\omega_m. \tag{11}$$

The non-dimensional damping ratio, ζ_m, is defined as the ratio of f_m over f_{mc},

$$\zeta_m = \frac{f_m}{f_{mc}} = \frac{f_m}{2\omega_m} = \frac{1}{2\tau_q\omega_m} + \frac{\tau_T\omega_m}{2}. \tag{12}$$

The system is at underdamped oscillation, critically damped oscillation or overdamped oscillation, respectively, when $\zeta_m < 1$, $\zeta_m = 1$ or $\zeta_m > 1$. By (10) and (12), we have an expression of $\lambda_{1,2}$ in terms of ζ_m and ω_m,

$$\lambda_{1,2} = \omega_m\left(-\zeta_m \pm \sqrt{\zeta_m^2 - 1}\right). \tag{13}$$

2.1. Underdamped oscillation

For this case ($\zeta_m < 1$), we have two complex conjugate λ_1, λ_2,

$$\lambda_{1,2} = \omega_m\left(-\zeta_m \pm \mathrm{i}\sqrt{1 - \zeta_m^2}\right). \tag{14}$$

Therefore

$$\Gamma_m(t) = \mathrm{e}^{-\zeta_m\omega_m t}\left(a_m \cos \omega_m t\sqrt{1-\zeta_m^2} + b_m \sin \omega_m t\sqrt{1-\zeta_m^2}\right). \tag{15}$$

After the determination of integration constants a_m and b_m by the initial conditions [Eq. (6)], we have

$$\Gamma_m(t) = \mathrm{e}^{-\zeta_m\omega_m t}\left(\phi_m \cos \omega_m t\sqrt{1-\zeta_m^2} + \frac{\psi_m + \zeta_m\omega_m\phi_m}{\omega_m\sqrt{1-\zeta_m^2}} \sin \omega_m t\sqrt{1-\zeta_m^2}\right), \tag{16}$$

which may be rewritten as

$$\Gamma_m(t) = A_m \mathrm{e}^{-\zeta_m\omega_m t}\sin(\omega_{dm}t + \varphi_{dm}). \tag{17}$$

Here,

$$A_m = \sqrt{\phi_m^2 + \left(\frac{\psi_m + \zeta_m\omega_m\phi_m}{\omega_{dm}}\right)^2}, \tag{18}$$

$$\omega_{dm} = \omega_m\sqrt{1-\zeta_m^2}, \tag{19}$$

$$\varphi_{dm} = \tan^{-1}\left(\frac{\phi_m\omega_{dm}}{\psi_m + \zeta_m\omega_m\phi_m}\right). \tag{20}$$

Therefore, the system is oscillating with the frequency ω_{dm} and an exponentially decaying amplitude $A_m\mathrm{e}^{-\zeta_m\omega_m t}$ [Eq. (7)]. Fig. 1 typifies the oscillatory pattern, for $\zeta_m = 0.1$, $\phi_m = 1.0$, $\psi_m = 0.0$ and $\omega_m = 1.0$. The wave behavior is still observed in the dual-phase-lagging heat conduction. However, the amplitude decays exponentially due to the damping of thermal diffusion. This differs very much from the classical heat conduction. $\zeta_m < 1$ forms the condition for the thermal oscillation of this kind.

Fig. 2 illustrates the variation of $\Gamma_m(t)$ with the time t when ψ_m is changed to 1.0 from 0. Γ_m is observed to be able of surpassing ϕ_m at some instants. Such phenomenon is caused by the no-vanishing initial time-rate change of the temperature and cannot appear in the classical heat conduction. The classical maximum and minimum principle is, therefore, not valid in the dual-phase-lagging heat conduction.

While $\Gamma_m(t)$ is oscillatory, it is not periodic because of the decaying amplitude. $\Gamma_m(t)$ oscillates in time with a fixed damped period T_{dm} given by

$$T_{dm} = \frac{2\pi}{\omega_{dm}}. \tag{21}$$

2.2. Critically damped oscillation

For this case, $\zeta_m = 1$. This requires, by Eq. (12),

$$\omega_m = \frac{1 \pm \sqrt{1 - (\tau_T/\tau_q)}}{\tau_T}. \tag{22}$$

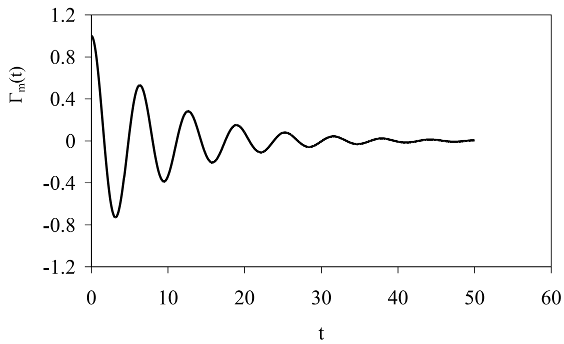

Fig. 1. Variation of $\Gamma_m(t)$ with the time t: $\zeta_m = 0.1$, $\omega_m = 1.0$, $\phi_m = 1.0$, $\psi_m = 0.0$.

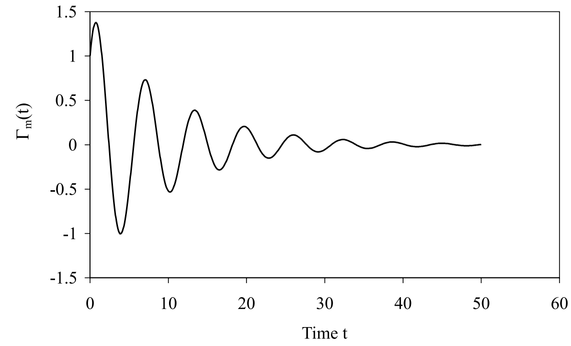

Fig. 2. Variation of $\Gamma_m(t)$ with the time t: $\zeta_m = 0.1$, $\omega_m = 1.0$, $\phi_m = 1.0$, $\psi_m = 1.0$.

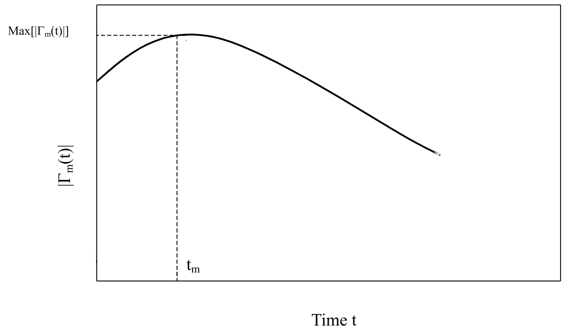

Fig. 3. $|\Gamma_m(t)|$ at the critically damped oscillation and $\psi_m^2 > -\omega_m\phi_m\psi_m$.

Therefore, the critically damped oscillation appears only when $\tau_T \leqslant \tau_q$. When the system is in the critically damped oscillation, we have two equal λ_1, λ_2. Therefore,

$$\Gamma_m(t) = a_m e^{-\omega_m t} + b_m t e^{-\omega_m t},$$

which becomes after determining the integration constants a_m and b_m by the initial conditions [Eq. (6)],

$$\Gamma_m(t) = e^{-\omega_m t}[\phi_m + (\psi_m + \omega_m\phi_m)t]. \tag{23}$$

Letting $d|\Gamma_m(t)|/dt = 0$ and analyzing the sign of $d^2|\Gamma_m(t)|/dt^2$, we obtain the maximal value of $|\Gamma_m(t)|$

$$\mathrm{Max}[|\Gamma_m(t)|] = \exp\left\{-\frac{\psi_m}{\psi_m + \omega_m\phi_m}\right\}\left|\phi_m + \frac{\psi_m}{\omega_m}\right| \tag{24}$$

at

$$t_m = \frac{\psi_m}{\omega_m(\psi_m + \omega_m\phi_m)}, \tag{25}$$

that is positive if

$$\psi_m^2 > -\omega_m\phi_m\psi_m.$$

This clearly requires that $\psi_m \neq 0$. Therefore, $|\Gamma_m(t)|$ decreases monotonically as t increases from 0 when

$$\psi_m^2 \leqslant -\omega_m\phi_m\psi_m.$$

This is very similar to that in the classical heat-conduction equation. When

$$\psi_m^2 > -\omega_m\phi_m\psi_m,$$

however, $|\Gamma_m(t)|$ first increases from ϕ_m to $\mathrm{Max}[|\Gamma_m(t)|]$ as t increases from 0 to t_m and then decreases monotonically (Fig. 3). Therefore, although the temperature field does not oscillate, its absolute value reaches the maximum value at $t = t_m > 0$ rather than the initial time instant $t = 0$.

2.3. Overdamped oscillation

For this case ($\zeta_m > 1$), we have

$$\lambda_{1,2} = \omega_m\left(-\zeta_m \pm \sqrt{\zeta_m^2 - 1}\right). \tag{26}$$

The solution of Eq. (7) is, thus, subject to the condition (6),

$$\Gamma_m(t) = \frac{e^{-\zeta_m\omega_m t}}{2\sqrt{\zeta_m^2-1}}\left[\left(\frac{\psi_m}{\omega_m} + \phi_m\left(\zeta_m + \sqrt{\zeta_m^2-1}\right)\right) \times e^{\omega_m t\sqrt{\zeta_m^2-1}} + \left(-\frac{\psi_m}{\omega_m} + \phi_m\left(-\zeta_m + \sqrt{\zeta_m^2-1}\right)\right) \times e^{-\omega_m t\sqrt{\zeta_m^2-1}}\right]. \tag{27}$$

Letting $d|\Gamma_m(t)|/dt = 0$ leads to two extreme points,

$$t_{m1} = 0 \tag{28}$$

and

$$t_{m2} = -\frac{1}{2\omega_m\sqrt{\zeta_m^2-1}} \times \ln\left[\frac{\zeta_m - \sqrt{\zeta_m^2-1}}{\zeta_m + \sqrt{\zeta_m^2-1}}\,\frac{(\psi_m/\omega_m) + \phi_m(\zeta_m + \sqrt{\zeta_m^2-1})}{(\psi_m/\omega_m) + \phi_m\left(\zeta_m - \sqrt{\zeta_m^2-1}\right)}\right], \tag{29}$$

with $\mathrm{Max1}[|\Gamma_m(t)|] = |\phi_m|$ and $\mathrm{Max2}[|\Gamma_m(t)|] = |\Gamma_m(t_{m2})|$, respectively. Therefore, $|\Gamma_m(t)|$ decreases monotonically from $t = 0$ when $t_{m2} = 0$ (very like that in classical heat conduction). When $t_{m2} > 0$, however, $|\Gamma_m(t)|$ first increases from $|\phi_m|$ to a maximal value $\mathrm{Max2}[|\Gamma_m(t)|]$ as t increases from 0 to t_{m2} and then decreases for $t \geqslant t_m$ (Fig. 4). There is no oscillation if $\zeta_m > 1$.

When $\tau_T > \tau_q$,

$$1 + \alpha\tau_T\frac{m^2\pi^2}{l^2} > 1 + \alpha\tau_q\frac{m^2\pi^2}{l^2} \geqslant 2\sqrt{\alpha\tau_q}\frac{m\pi}{l}.$$

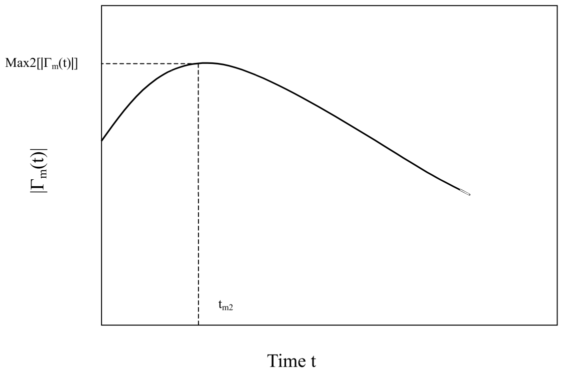

Fig. 4. $|\Gamma_m(t)|$ are the overdamped oscillation and $t_{m2} > 0$.

This, with Eq. (12), yields

$$\zeta_m = \frac{1 + \tau_T \alpha (m^2\pi^2/l^2)}{2\sqrt{\alpha\tau_q}(m\pi/l)} > 1.$$

Therefore the system is always at the overdamped oscillation if $\tau_T > \tau_q$. Consequently, there is no thermal oscillation.

$\zeta_m = 1.0$ separates the underdamped modes from the overdamped modes. Applying $\zeta_m < 1$ in Eq. (12) yields the region of m where the underdamped modes can occur:

$$\begin{aligned} &\frac{l}{\pi C_1} < m < \frac{l}{\pi C_2} \quad \text{if } \tau_q > \tau_T > 0, \\ &m > \frac{l}{\pi C} \quad \text{if } \tau_q > \tau_T = 0. \end{aligned} \tag{30}$$

Here C_1, C_2 and C are the relaxation distances [21] and defined by

$$\begin{aligned} C_1 &= \sqrt{\alpha\tau_T}\left(\sqrt{\frac{\tau_q}{\tau_T}} + \sqrt{\frac{\tau_q}{\tau_T} - 1}\right), \\ C_2 &= \sqrt{\alpha\tau_T}\left(\sqrt{\frac{\tau_q}{\tau_T}} - \sqrt{\frac{\tau_q}{\tau_T} - 1}\right), \\ C &= 2\sqrt{\alpha\tau_q}. \end{aligned}$$

Therefore, the thermal oscillation occurs only for the modes between $l/\pi C_1$ and $l/\pi C_2$ for the case of $\tau_q > \tau_T > 0$. This is different from the thermal wave in the hyperbolic heat conduction where the oscillation appears always for the high-order modes [21].

The behavior of an individual temperature mode discussed above also represents the entire thermal response if $\phi(x) = A\sin(m\pi x/l)$ and $\psi(x) = B\sin(m\pi x/l)$ with A and B as constants. For the general case, a change $\Delta\Gamma_m(t)$ in the mth mode would lead to a change $\Delta\Gamma_m(t)\sin(m\pi x/l)$ in $T(x,t)$ because

$$T(x,t) = \sum_{m=1}^{\infty} \Gamma_m(t) \sin\frac{m\pi x}{l}.$$

3. Resonance

For the dual-phase-lagging heat conduction, the amplitude of the thermal wave may become exaggerated if the oscillating frequency of an externally applied heat source is at the resonance frequency.

Consider a heat source in the system (1) in the form of

$$S(x,t) = Qg(x)\,e^{i\Omega t}.$$

Here Q, independent of x and t, is the strength, $g(x)$ the spanwise distribution, and Ω the oscillating frequency. Expand $T(x,t)$ and $g(x)$ by the Fourier sine series,

$$T(x,t) = \sum_{m=1}^{\infty} \Gamma_m(t) \sin(\beta_m x), \tag{31}$$

$$g(x) = \sum_{m=1}^{\infty} D_m \sin(\beta_m x), \tag{32}$$

where $\Gamma_m(t)$ and β_m are defined in the last section, and

$$D_m = \frac{2}{l}\int_0^l g(x)\sin(\beta_m x)\,dx. \tag{33}$$

Such a $T(x,t)$ in (31) automatically satisfies the boundary conditions in (1). Substituting Eqs. (31) and (32) into Eq. (1) and making use of the orthogonality of the set $\sin(\beta_m x)$ yield

$$\begin{aligned} &\ddot{\Gamma}_m(t) + 2\zeta_m\omega_m\dot{\Gamma}_m(t) + \omega_m^2\Gamma_m(t) \\ &\quad = \frac{QD_m\alpha}{k\tau_q}(1 + i\Omega\tau_q)\,e^{i\Omega t}, \end{aligned} \tag{34}$$

whose solution is readily obtained as

$$\Gamma_m(t) = B_m\,e^{(\Omega t + \varphi_m)i}. \tag{35}$$

Here,

$$B_m = \frac{QD_m\alpha}{k\omega_m}B_{\Omega_m^*}, \tag{36}$$

$$B_{\Omega_m^*} = \frac{\eta_m + i\Omega_m^*}{\sqrt{(1 - \Omega_m^* 2)^2 + 4\zeta_m^2\Omega_m^* 2}}, \tag{37}$$

$$\tan^{-1}(\varphi_m) = -\frac{2\zeta_m\Omega_m^*}{1 - \Omega_m^* 2}, \tag{38}$$

$$\eta_m = \frac{1}{\sqrt{\alpha\tau_q}\beta_m}, \tag{39}$$

$$\Omega_m^* = \frac{\Omega}{\omega_m}. \tag{40}$$

For the resonance, $|B_{\Omega_m^*}|^2$ reaches its maximum value. Note that, by (37),

$$|B_{\Omega_m^*}|^2 = \frac{\eta_m^2 + \Omega_m^{*2}}{(1 - \Omega_m^{*2})^2 + 4\zeta_m^2\Omega_m^{*2}}. \tag{41}$$

Therefore, the resonance requires

$$\frac{\partial|B_{\Omega_m^*}|^2}{\partial\Omega_m^{*2}} = 0,$$

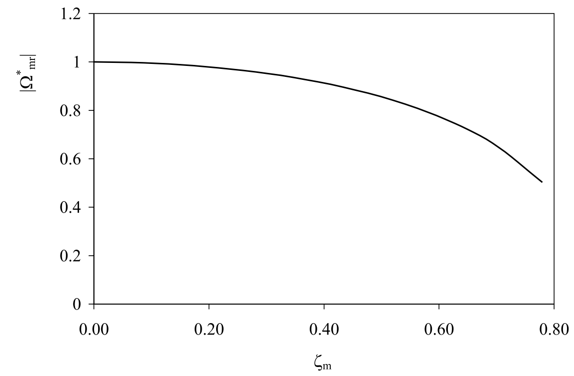

Fig. 5. Variation of $|\Omega_{mr}^*|$ with ζ_m at $\eta_m = 1.0$.

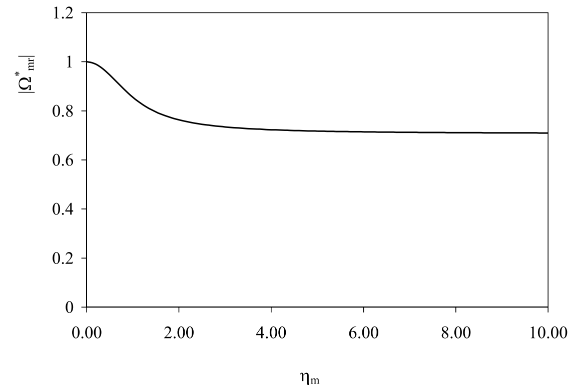

Fig. 6. Variation of $|\Omega_{mr}^*|$ with η_m at $\zeta_m = 0.5$.

which yields, by noting also that $\Omega_m^* \geqslant 0$,

$$\Omega_{mr}^{*^2} = -\eta_m^2 + \sqrt{\left(1+\eta_m^2\right)^2 - 4\zeta_m^2\eta_m^2}, \tag{42}$$

which Ω_{mr}^* stands for the external source frequency at resonance. As Ω_{mr}^* must be real, we have another condition for the resonance in addition to (42),

$$\left(1+\eta_m^2\right)^2 - 4\zeta_m^2\eta_m^2 > \eta_m^4. \tag{43}$$

The variation of Ω_{mr}^* with the ζ_m and η_m is shown in Figs. 5 and 6. It is observed that Ω_{mr}^* decreases as the damping parameter ζ_m and the phase-lagging parameter η_m increase. Fig. 7 illustrates the variation of $|B_{\Omega_m^*}|$ with Ω_m^* and ζ_m at $\eta_m = 1$. For $\zeta_m = 0.9$, (43) cannot be satisfied. Therefore, there is no resonance when $\zeta_m = 0.9$ at $\eta_m = 1$ (Fig. 7).

4. Concluding remarks

The thermal oscillation described by the dual-phase-lagging heat-conduction equation is characterized by ζ_m

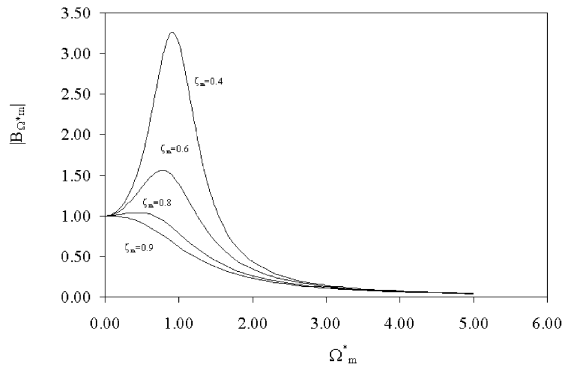

Fig. 7. Variation of $|B_m^{\Omega*}|$ with Ω_m^* and $\zeta_m : \eta_m = 1$.

defined by Eq. (12) and can be underdamped ($\zeta_m < 1$), critically damped ($\zeta_m = 1$), or overdamped ($\zeta_m > 1$). The underdamped oscillating modes are in a region defined by Eq. (30). When the phase lag of the temperature gradient τ_T is larger than that of the heat flux τ_q, the system is overdamped such that there is no oscillation. The evolution of temperature field differs from that of classical heat conduction even when the system is at the critically damped or overdamped state. The temperature does not decay monotonically in general, when there is an initial time-rate change of the temperature in particular. The classical maximum and minimum principle is thus invalid for the dual-phase-lagging heat conduction.

Eqs. (42) and (43) are the conditions for the appearance of thermal resonance.

Acknowledgements

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